

A full-wave integral equation method including accurate wide-frequency-band wire models for WPT coils

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A full-wave surface integral equation method is used for modeling coils in a wide frequency range. This is of high importance in, for example, the analysis of resonant wireless power transfer (WPT) links. Previous contributions of the field are extended herein by incorporating a 2D model (using finite elements here) of the current distribution within the wire. This yields a numerical approximation of the non-local impedance boundary condition on the wire surface that holds true at any frequency. The 2D eddy-current model has to be evaluated only twice at one frequency, which makes the scheme computationally efficient. The method accurately predicts the ohmic loss as shown by an example.

Index Terms—coil, finite elements, impedance boundary condition, integral equation, wireless power transfer

I. INTRODUCTION

THE ELECTROMAGNETIC (EM) modeling of coils under ac excitation is still a challenging problem in spite of the several attempts presented in the literature, especially in the case when the joule loss in the wire is to be accurately calculated. The main difficulty lies in the large aspect ratio of the coil size and wire diameter. Furthermore, one has an eddy-current problem in the conductor whereas a full-wave model might apply in the surrounding dielectric. Such problems typically occur in the analysis of resonant wireless power transfer (WPT) links.

Integral equation (IE) methods have shown a good performance in coil modeling [1]. A full-wave surface IE method with special attention to WPT modeling has been presented in [2], using the Leontovich surface impedance boundary condition (SIBC) [3]. More sophisticated SIBC approximations are known, e.g., the analytic approach of [4]. In the present contribution, the formulation in [2] is extended by a novel approximation of the SIBC based on a 2D analysis of the eddy-current distribution over the wire cross section. A 2D finite element method (FEM) is used here, but other numerical techniques could also be applied.

II. THE INTEGRO-DIFFERENTIAL EQUATION MODEL

Let us consider a coil wire made of homogeneous, non-magnetic conductor with electric conductivity κ , that stands

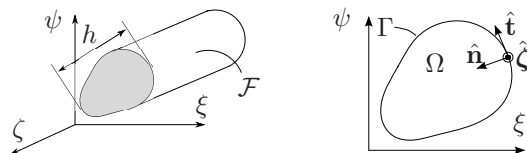


Fig. 1. Local coordinate system and the 2D model domain Ω of the wire cross-section with the boundary Γ . $\hat{\mathbf{t}}$, $\hat{\mathbf{n}}$ and $\hat{\boldsymbol{\zeta}} = \hat{\mathbf{t}} \times \hat{\mathbf{n}}$ are the tangential, normal and longitudinal unit vectors, respectively. h is the largest linear extent of the wire cross-section; \mathcal{F} refers to the wire surface.

in a homogeneous dielectric medium with permittivity ε . The wire can form a loop or coil with arbitrary shape. Let the radius of curvature be much larger than the largest linear extent of the wire cross-section, but no constraint is set to the gap between adjacent turns. A section of the wire is shown in Fig. 1 along with a local Cartesian coordinate system $\{\xi, \psi, \zeta\}$. We consider a time-harmonic excitation with angular frequency ω .

One assumes a slow variation of the volume current density $\mathbf{J} = \kappa \mathbf{E}$ along the wire (i.e., in the local ζ direction) in the sense that $\partial/\partial\zeta$ can be neglected when studying the variation over the cross-section. Thus the EM field in the wire obeys a partial differential equation (PDE) over the 2D domain Ω (Fig. 1). The key idea of the proposed method is to find the particular solution of this PDE with respect to the boundary condition (BC) on the wire surface. The latter links the eddy-current PDE within the conductor to the IE model being valid in the surrounding dielectric. The EM field can be described in terms of the longitudinal components of the electric E^ζ and magnetic H^ζ fields at any arbitrary cross-section of the wire:

- a) By assuming that $H^\zeta = 0$ (transverse magnetic, TM case), one has the Helmholtz's equation [5] for E^ζ as

$$\nabla^2 E^\zeta(\xi, \psi) - \gamma_c^2 E^\zeta(\xi, \psi) = 0 \quad \text{on } \Omega \quad (1)$$

with the propagation constant $\gamma_c = \sqrt{j\omega\mu_0\kappa}$. In this case, the tangential component of the magnetic field imposes a Neumann boundary condition for E^ζ on Γ :

$$\left. \frac{\partial E^\zeta}{\partial n} \right|_\Gamma = -j\omega\mu_0 H^t \Big|_\Gamma. \quad (2)$$

- b) By assuming that $E^\zeta = 0$ (transverse electric, TE case), the same PDE is written for H^ζ :

$$\nabla^2 H^\zeta(\xi, \psi) - \gamma_c^2 H^\zeta(\xi, \psi) = 0 \quad \text{on } \Omega, \quad (3)$$

together with a Dirichlet boundary condition for H^ζ on Γ . The tangential electric field on the surface is then

$$E^t \Big|_\Gamma = \frac{1}{\kappa} \left. \frac{\partial H^\zeta}{\partial n} \right|_\Gamma. \quad (4)$$

In the cases of a) and b), one has the non-local SIBC expressions

$$E^\zeta \Big|_\Gamma = \mathcal{Z}_S^{\zeta t} \left\{ H^t \Big|_\Gamma \right\}, \text{ and } E^t \Big|_\Gamma = \mathcal{Z}_S^{t\zeta} \left\{ H^\zeta \Big|_\Gamma \right\}. \quad (5)$$

The operators $\mathcal{Z}_S^{\zeta t}$ and $\mathcal{Z}_S^{t\zeta}$ represent a 2D eddy-current boundary value problem and they are evaluated by the FEM. Note that only 2 FEM simulations are needed at one frequency.

In the dielectric, the \mathbf{A} - Φ formulation is used, i.e., the magnetic vector and electric scalar potentials are written as

$$\begin{aligned} \mathbf{A}(\mathbf{r}) &= \mu_0 \int_{\mathcal{F}} g(\mathbf{r}, \mathbf{r}') \mathbf{J}_S(\mathbf{r}') d\mathcal{F}' \\ \Phi(\mathbf{r}) &= \frac{1}{\varepsilon_0} \int_{\mathcal{F}} g(\mathbf{r}, \mathbf{r}') \sigma(\mathbf{r}') d\mathcal{F}' \end{aligned} \quad (6)$$

with $g(\mathbf{r}, \mathbf{r}') = \exp(-j\omega\sqrt{\mu_0\varepsilon_0}|\mathbf{r} - \mathbf{r}'|)/(4\pi|\mathbf{r} - \mathbf{r}'|)$ being the free-space Green's function [5], the sources (the surface current density \mathbf{J}_S and surface charge density σ) are defined on the wire surface \mathcal{F} . The potentials provide the electric field as

$$\mathbf{E} = -\nabla\Phi - j\omega\mathbf{A}. \quad (7)$$

The sources satisfy the charge conservation law

$$0 = \nabla \cdot \mathbf{J}_S + j\omega\sigma. \quad (8)$$

Note that the current is represented as a surface current density \mathbf{J}_S in (6) and (8), whereas a volume current density \mathbf{J} is used in the eddy-current model in the wire. \mathbf{J}_S is approximately linked to \mathbf{J} via an integration in the direction normal to the surface. \mathbf{J}_S can be shown to have a direct relation with the tangential components of the magnetic field on the surface [5]:

$$H^t \Big|_\Gamma = J_S^\zeta, \quad H^\zeta \Big|_\Gamma = -J_S^t. \quad (9)$$

By using (7) and (9), one finally sees that the integral equations (6), the charge conservation law (8) and the SIBCs (5) together form a coupled system of integro-differential equations that can be solved for a given excitation, e.g., an imposed current at the wire terminals.

III. DISCRETIZATION FOR A TEST CASE

A circular loop and a helical coil are made of a cylindrical copper wire as shown in Fig. 2, with parameters $\kappa = 57 \text{ MS/m}$, $a = 0.89 \text{ mm}$, $d = 3a$, $h = 12a$ and $R = 111 \text{ mm}$.

In the integro-differential equation system (5)-(9) \mathbf{J}_S and σ are discretized by using piece-wise constant basis functions defined by a rectangular grid on the wire surface \mathcal{F} with grid-lines parallel with the unit vectors $\hat{\zeta}$ and \hat{t} . The cross-section of the wire is thus approximated by a regular m -sided polygon. The E^ζ , E^t and H^ζ , H^t components on \mathcal{F} are also approximated as piece-wise constant, i.e., the operators in (5) take the form

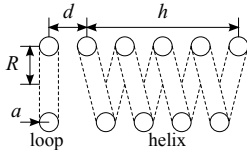


Fig. 2. The test configuration

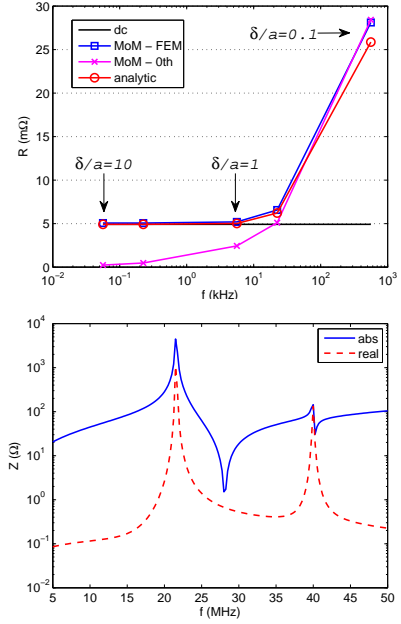


Fig. 3. Loop impedance at low ($\delta \approx a$) and high frequencies. “MoM - 0th” refers to the integral equation scheme with Leontovich SIBC, “MoM - FEM” stands for the proposed method, whereas “analytic” uses the closed-form solution for a straight wire in terms of Bessel functions [5]. “MoM - 0th” and “MoM - FEM” tend to be equivalent with increasing frequency.

of m -by- m matrices. The loop impedance is calculated with $m = 8$ and 21 segments per turn along ζ , i.e., with 840 surface elements in total. Finally, the method of moments is used with point collocation to obtain a system of algebraic equations. The results agree well with the expectations as shown in Fig. 3.

IV. CONCLUSION

By means of a 2D FEM model of the volume current density in the wire, a surface IE formulation is extended by a non-local SIBC that is not limited to the small skin-depth case and correctly models proximity effects. More details on the method along with a thorough validation will be given in the full paper.

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